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A Direct Analytical Approach for Solving Surface and Subsurface Water Flow on a Hillslope

Ping-Cheng Hsieh* Ching-Ya Tang Siou-Yi Hu

ABSTRACT Surface water flow passing over unplanted ground and vegetated ground is delineated analytically. The velocity distribution of surface and subsurface water flow is directly solved by the simplified Navier-Stokes equations and Biot's theory of poroelasticity. The velocity will increase with increasing water depth, slope and porosity of the vegetation-layer. Higher permeabilities of the soil layer will increase the velocity inside the soil, but it has little influence on the velocity above the ground. The mean velocity of the overland flow passing over a vegetated or non-vegetated hillslope is compared with those calculated by the Darcy-Weisbach (D-W) formula and the kinematic-wave (K-W) equation. For the case of unplanted ground, the result of the proposed approach is less than that of the K-W equation but larger than that of the D-W formula. For the case of emergent vegetation, the mean velocity of the D-W formula reaches its minimum when the porosity of the vegetation layer is larger than 0.94 and the surface water depth is less than 0.05 m.

Key Words : Surface water, laminar flow, vegetation, porous media flow, poroelasticity.

1. Introduction

In conventional studies on overland flow, simple empirical formulas, such as the Manning's formula, are usually employed to estimate the flow velocity. The determination of the coefficient of surface roughness is, however, up to the users' judgment based on their knowledge or experience. Izzard (1946) is a pioneer scholar to conduct hydraulic experiments on runoff and concluded that when the product of rainfall intensity *i* (in units of in./hr) and length of overland flow (in units of ft) is less than 500, the flow regime is laminar. Horton (1945) advocated that the overland flow should be separated into laminar flow and turbulent flow. Emmett (1970) conducted many overland flow experiments and found that artificially uniform flow in the lab could be turbulent or laminar depending on the roughness or smoothness of the ground surface. He also realized the difficulty of simulation of natural ground in laboratory. According to kinematic wave theory, Ragan and Duru (1972) estimated the concentration time using nomographs. Henderson and Wooding (1964) simplified Lighthill and Whitham's (1955) work and applied the method of characteristics to obtain the analytical solution of overland flow down a slope.

Wooding (1965) introduced the concept of a V-shaped watershed. Woolhiser (1969) proposed an overland flow model for a converging surface. Kibler and Woolhiser (1970) addressed a cascading style of hydrologic model. All these authors conceptualized the complicated watershed first and then solved the kinematic-wave equation by numerical methods. Later on, Singh (1997) also discussed the application of kinematic-wave model to the problem of water resources. As to these researches about the analysis of flow through vegetation, their focus is mostly on the resistance force acting upon the water body and thus on the nature of the velocity profile, but their works did not present a closed form solution of the vertical velocity profile used by other models, such as those proposed by Petryk and Bosmajian (1975), Kadlec (1990), Wu et al. (1999), and Kutija and Hong (1996).

Based on the abovementioned comments, this study selected the Darcy-Weisbach friction loss formula and the kinematic-wave equation for comparison. The former formula is an empirical formula lacking systematic theory and the latter is an over-simplified result of equation of motion. Although Hsieh and Bolton (2007) proposed a more theory-based research to discuss the surface water flow, they used an *indirect* method by degenerating their

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solutions into the two cases of unplanted ground and emergent vegetation. Additionally, their use of the Kozeny-Carman relation, which is commonly used for soils to describe the vegetation permeability, is not quite adequate. Hence, a *direct* analytical approach was employed and the permeability of vegetation layer was evaluated by Kaviany's (1991) formula instead in this study.

2. FORMULATION

1. Governing Equations

(1) Fully-developed steady homogeneous water flow

Considering a fully developed flow over unplanted ground (see Fig. 1 (a)),

$$\frac{\partial u_1}{\partial x} = 0 \tag{1}$$

the velocity can be found from the simplified equation of momentum

$$\text{x dir.: } \mu \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p_1}{\partial x} + \rho g \sin\theta = 0 \tag{2}$$

$$\text{y dir.: } \frac{\partial p_1}{\partial y} + \rho g \cos\theta = 0 \tag{3}$$

where u_1 = flow velocity in $\boldsymbol{x}$ direction of homogeneous water layer denoted by Region 1, p_1 = fluid pressure of Region 1, ρ = density of fluid, g = gravitational acceleration, μ = dynamic viscosity of fluid, and θ = angle of the inclined plane.

(2) Governing equations of laminar poroelastic media flow

Referring to Song and Huang (2000), the unsteady laminar poroelastic media flow in vegetation layer denoted by Region 2 and soil layer denoted by Region 3 can be described by Biot's theory of poroelasticity (1956) as follows:

$$\nabla\cdot\boldsymbol{\sigma}_\mathbf{s} = (1-n)\rho_s \frac{\partial^2 \mathbf{d}}{\partial t^2} - \frac{\mu n^2}{k_p}\left(\frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{d}}{\partial t}\right) \tag{4}$$

$$\nabla\cdot\boldsymbol{\sigma}_\mathbf{f} = n\rho \frac{\partial^2 \mathbf{D}}{\partial t^2} + \frac{\mu n^2}{k_p}\left(\frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{d}}{\partial t}\right) \tag{5}$$

with

$$\boldsymbol{\sigma}_\mathbf{s} = \boldsymbol{\tau}_\mathbf{s} - (1-n)p\mathbf{I} \tag{6}$$

$$\boldsymbol{\tau}_\mathbf{s} = 2G\mathbf{e} + \lambda(\nabla\cdot\mathbf{d})\mathbf{I} \tag{7}$$

$$\mathbf{e} = \frac{1}{2}(\nabla\mathbf{d} + (\nabla\mathbf{d})^T) \tag{8}$$

$$\boldsymbol{\sigma}_\mathbf{f} = -np\mathbf{I} + n\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) \tag{9}$$

$$\mathbf{u} = \frac{\partial \mathbf{D}}{\partial t} \tag{10}$$

Where, $\boldsymbol{\sigma}_\mathbf{s}$ = total stress tensor of the solid, $\boldsymbol{\tau}_\mathbf{s}$ = effective stress tensor of the solid; $\mathbf{e}$ = strain tensor of the solid, $\boldsymbol{\sigma}_\mathbf{f}$ = stress tensor of the fluid, $\mathbf{d}$ and $\mathbf{D}$ = displacements of the solid and fluid, respectively, $\mathbf{u}$ = pore velocity of the fluid, p = pore fluid pressure, ρ_s = density of the solid, n = porosity, k_p = coefficient of specific permeability, G and λ = Lame constants of elasticity, t = time, T = transpose of the matrix ,and $\mathbf{I}$ = identity matrix.

Assuming that the vegetation layer is homogeneous and isotropic, the plant in Region 2 is in a steady state and its shape is not recoverable after bending by constant stream flow so that Eqs. (4) and (6) can be eliminated. Hence, we can obtain the continuity equation

$$\frac{\partial u_2}{\partial x} = 0 \tag{11}$$

and the momentum equations from Eq. (5)

$$\mu \frac{\partial^2 u_2}{\partial y^2} - \frac{\mu n_2}{k_{p2}} u_2 - \frac{\partial p_2}{\partial x} + \rho g \sin\theta = 0 \tag{12}$$

$$\frac{\partial p_2}{\partial y} + \rho g \cos\theta = 0 \tag{13}$$

in which u_2 and p_2 = average pore fluid velocity in the $\boldsymbol{x}$ direction and pore fluid pressure in Region 2, respectively; n_2 and k_{p2} = porosity and the coefficient of specific permeability in Region 2, respectively.

Moreover, assuming that the soil grains are not moveable by the stream flow, the equations of continuity and momentum of the porous soil layer, i.e. Region 3 (see Fig. 1(b), will become

$$\frac{\partial u_3}{\partial x} = 0 \tag{14}$$

$$\mu\frac{\partial^2 u_3}{\partial y^2}-\frac{\mu n_3}{k_{p3}}u_3-\frac{\partial p_3}{\partial x}+\rho\, g\sin\theta=0 \quad (15)$$

$$\frac{\partial p_3}{\partial y}+\rho\, g\cos\theta=0 \quad (16)$$

2. Boundary Conditions

According to Deresiewicz and Skalak (1963), three boundaries-- (1) free surface ($y=h_1$ or $y=h_2$), (2) the surface of porous soil layer ($y=0$), and (3) soil layer/impervious rock interface ($y=-H$) are required to satisfy the following boundary conditions.

(1) Unplanted ground

① At the free surface: The stress of the homogeneous water can be expressed as

$$\boldsymbol{\sigma}_{\mathbf{f}}=-p\mathbf{I}+\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T) \quad (17)$$

According to Eq. (17), we can obtain the boundary conditions at the free surface as follows:

- Continuity of fluid stress in normal (y) direction

$$\sigma_{fyy1}=-p_1+2\mu\frac{\partial v_1}{\partial y}=p_{atm}=0 \quad (18)$$

After applying the assumption of fully developed flow and gauge pressure, we get

$$p_1=0 \quad (19)$$

- Continuity of fluid stress in tangential (x) direction

$$\sigma_{fxy1}=\mu(\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x})=\mu\frac{\partial u_1}{\partial y}=0 \quad (20)$$

② At the surface of porous soil layer:

- Continuity of velocity in tangential (x) direction

$$u_1=n_3u_3 \quad (21)$$

- Continuity of fluid stress in tangential (x) direction

$$\sigma_{fyx1}=\frac{\sigma_{fyx3}}{n_3} \quad (22)$$

and according to Eq. (9), Eq. (22) becomes

$$\frac{\partial u_1}{\partial y}=\frac{\partial u_3}{\partial y} \quad (23)$$

- Continuity of fluid stress in normal (y) direction

$$\sigma_{fyy1}=\frac{\sigma_{fyy3}}{n_3} \quad (24)$$

and according to Eq.(9), Eq.(24) becomes

$$p_1=p_3 \quad (25)$$

③ At the soil layer/impervious rock interface:

- Non-slip condition in tangential (x) direction

$$u_3=0 \quad (26)$$

(2) Emergent vegetation

① At the free surface:

- Continuity of fluid stress in normal (y) direction

$$\sigma_{fyy2}=-p_2+2\mu\frac{\partial v_2}{\partial y}=p_{atm}=0 \quad (27)$$

After applying the assumption of fully developed flow and gauge pressure, we get

- $p_2=0$ (28)

- Continuity of fluid stress in tangential (x) direction

$$\sigma_{fxy2}=\mu(\frac{\partial u_2}{\partial y}+\frac{\partial v_2}{\partial x})=\mu\frac{\partial u_2}{\partial y}=0 \quad (29)$$

② At the interface between the vegetation layer and soil layer

- Continuity of velocity in tangential (x) direction

$$n_2u_2=n_3u_3 \quad (30)$$

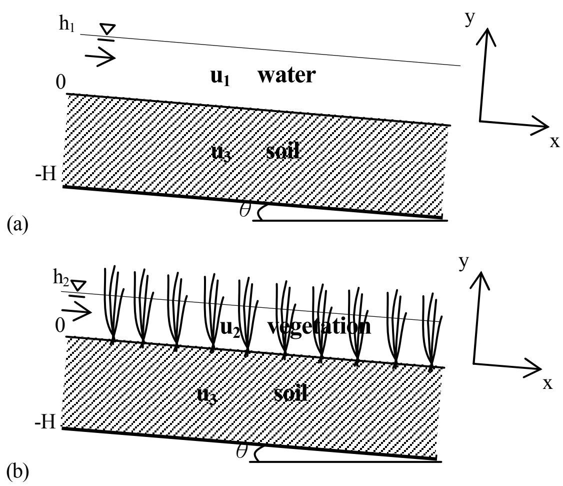


Fig.1 Definition sketch of flow over (a) unplanted ground and (b) emergent vegetation

- Continuity of fluid stress in tangential (x) direction

$$\frac{\sigma_{fyx2}}{n_2}=\frac{\sigma_{fyx3}}{n_3} \quad (31)$$

and according to Eq.(9), Eq. (31) becomes

$$\frac{\partial u_2}{\partial y}=\frac{\partial u_3}{\partial y} \quad (32)$$

- Continuity of fluid stress in normal (y) direction

$$\frac{\sigma_{fyy2}}{n_2}=\frac{\sigma_{fyy3}}{n_3} \quad (33)$$

and according to Eq. (9), Eq. (33) becomes

$$p_2=p_3 \quad (34)$$

③ At the soil layer/impervious rock interface:

- Non-slip condition in tangential (x) direction

$$u_3=0 \quad (35)$$

3. Analytical Solutions

(1) Unplanted ground

From Eq. (3), we have

$$p_1=-\rho g\cos\theta\cdot y+f(x) \quad (36)$$

and with boundary condition (19) we find $\partial p_1/\partial x=0$. Thus Eq. (2) becomes

$$\mu\frac{\partial^2 u_1}{\partial y^2}+\rho g\sin\theta=0 \quad (37)$$

Similarly, p_3 is only a function of y, and Eq. (15) becomes

$$\mu\frac{\partial^2 u_3}{\partial y^2}-\frac{\mu n_3}{k_{p3}}u_3+\rho g\sin\theta=0 \quad (38)$$

The general solution of Eqs. (37) and (38) is

$$u_1=-\frac{1}{2}\frac{\rho g sin\theta}{\mu}y^2+C_1y+C_2, \quad 0<y<h_1 \quad (39)$$

$$u_3=\frac{\rho g sin\theta k_{p3}}{n_3\mu}+C_3 sinh(\sqrt{n_3/k_{p3}}y)+C_4 cosh(\sqrt{n_3/k_{p3}}y),\ -H<y<0 \quad (40)$$

In which, $C_1\sim C_4$ are undetermined coefficients. Substituting Eqs. (39) and (40) into boundary conditions (20), (21), (23) and (26), the coefficients can be obtained, and the velocity distribution of u_1 and u_3 becomes

$$u_1=\frac{\rho_w g sin\theta}{\mu}[-\frac{1}{2}y^2+h_1y+k_{p3}+\frac{h_1\sqrt{n_3k_{p3}}sinh(\sqrt{n_3/k_{p3}}H)-k_{p3}}{cosh(\sqrt{n_3/k_{p3}}H)}],\ 0<y<h_1 \quad (41)$$

$$u_3=\frac{\rho_w g sin\theta}{\mu}[\frac{k_{p3}}{n_3}+h_1\sqrt{\frac{k_{p3}}{n_3}}sinh(\sqrt{n_3/k_{p3}}y)+\frac{h_1\sqrt{n_3k_{p3}}sinh(\sqrt{n_3/k_{p3}}H)-k_{p3}}{n_3 cosh(\sqrt{n_3/k_{p3}}H)}\cdot cosh(\sqrt{n_3/k_{p3}}y)],\ -H<y<0 \quad (42)$$

(2) Emergent vegetation

From Eq. (13), we have

$$p_2=-\rho g\cos\theta\cdot y+q(x) \quad (43)$$

and with boundary condition (27) we find $\partial p_2/\partial x=0$. Thus Eq. (12) becomes

$$\mu\frac{\partial^2 u_2}{\partial y^2}-\frac{\mu n_2}{k_{p2}}u_2+\rho g\sin\theta=0 \quad (44)$$

Similarly, p_3 is only a function of y, and Eq. (15) becomes

$$\mu\frac{\partial^2 u_3}{\partial y^2}-\frac{\mu n_3}{k_{p3}}u_3+\rho g\sin\theta=0 \quad (45)$$

The general solution of Eqs. (44) and (45) is

$$u_2=\frac{\rho_w g\sin\theta k_{p2}}{n_2\mu}+C_5\sinh\left(\sqrt{n_2/k_{p2}y}\right)+C_6\cosh\left(\sqrt{n_2/k_{p2}y}\right),0<y<h_2 \quad (46)$$

$$u_3=\frac{\rho_w g\sin\theta k_{p3}}{n_3\mu}+C_7\sinh\left(\sqrt{n_3/k_{p3}y}\right)+C_8\cosh\left(\sqrt{n_3/k_{p3}y}\right),-H<y<0 \quad (47)$$

Where $C_5\sim C_8$ are undetermined coefficients. Substituting Eqs. (46) and (47) into boundary conditions (29), (30), (32) and (35), the coefficients can be obtained, and the velocity distribution of u_1 and u_3 comes to

$$u_2=\frac{\rho_w g\sin\theta}{\mu}\cdot\left\{\frac{k_{p2}}{n_2}+\frac{\sqrt{\frac{n_3}{k_{p3}}}\left[(k_{p2}-k_{p3})\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+k_{p3}\right]\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}y\right)}{\sqrt[n_3]{\frac{n_2}{k_{p2}}}\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+\sqrt[n_2]{\frac{n_3}{k_{p3}}}\cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)}\right.$$
$$\left.-\frac{\sqrt{\frac{n_3}{k_{p3}}}\left[(k_{p2}-k_{p3})\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+k_{p3}\right]\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}y\right)}{\sqrt[n_3]{\frac{n_2}{k_{p2}}}\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+\sqrt[n_2]{\frac{n_3}{k_{p3}}}\cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)}\right\}\ 0<y<h_2 \quad (48)$$

$$u_3=\frac{\rho_w g\sin\theta}{\mu}\cdot\left\{\frac{k_{p3}}{n_3}\right.$$
$$+\frac{\sqrt{\frac{n_2}{k_{p2}}}\left[(k_{p2}-k_{p3})\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+k_{p3}\right]\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_3}{k_{p3}}}y\right)}{\sqrt[n_3]{\frac{n_2}{k_{p2}}}\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+\sqrt[n_2]{\frac{n_3}{k_{p3}}}\cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)}+\left\{\frac{1}{n_3}(k_{p2}-k_{p3})\cosh\left(\sqrt{n_3/k_{p3}}\,y\right)\right.$$
$$\left.+\frac{\frac{n_2}{n_3}\sqrt{\frac{n_3}{k_{p3}}}\left[(k_{p3}-k_{p2})\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)-k_{p3}\right]\cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}y\right)}{\sqrt[n_3]{\frac{n_2}{k_{p2}}}\sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\sinh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+\sqrt[n_2]{\frac{n_3}{k_{p3}}}\cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)\cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)}\right\}\ -\mathrm{H}<y<0 \quad (49)$$

Then the mean velocities $\overline{u}_1$ and $\overline{u}_2$ could be obtained, respectively, as follows:

$$\overline{u_1}=\frac{\int_0^{h_1}u_1dy}{h_1}=\frac{\rho g\,sin\,\theta}{\mu}\left[\frac{1}{3}h_1^{\,2}+k_{p3}+\frac{h_1\sqrt{n_3k_{p3}}\,sinh(\sqrt{n_3/k_{p3}}H)-k_{p3}}{cosh(\sqrt{n_3/k_{p3}}H)}\right] \quad (50)$$

$$\overline{u_2}=\frac{\int_0^{h_2}u_2dy}{h_2}=\frac{\rho g\,sin\,\theta}{\mu\cdot h_2}\cdot\left\{\frac{k_{p2}}{n_2}\cdot h_2-\frac{\sqrt{\frac{k_{p2}}{n_2}}\sqrt{\frac{n_3}{k_{p3}}}\left[(k_{p2}-k_{p3})cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+k_{p3}\right]sinh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)}{\sqrt[n_3]{\frac{n_2}{k_{p2}}}\,sinh\left(\sqrt{\frac{k_{p2}}{n_2}}h_2\right)sinh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)+\sqrt[n_2]{\frac{n_3}{k_{p3}}}\,cosh\left(\sqrt{\frac{n_2}{k_{p2}}}h_2\right)cosh\left(\sqrt{\frac{n_3}{k_{p3}}}H\right)}\right. \quad (51)$$

Since specific permeability is strongly related to porosity and the porosity of vegetation layer is easier to measure or calculate than the specific permeability of vegetation layer, Kaviany's (1991) formula is introduced as shown in Eq. (52).

$$k_{p2}=\frac{n_2^{\ 3}}{80(1-n_2)^2}d_c^2 \quad (52)$$

Where d_c is the average diameter of the stems.

(3) Non-dimensionalization of velocity

Referring to Buckingham's π -method, we can obtain the following dimensionless flow velocities.

$$\frac{u_1}{u_{1\max}}=f\left(\frac{y}{h_1};\frac{\rho^2\mathrm{h}_1^3 g\sin\theta}{\mu^2},\sqrt{\frac{n_3}{k_{p3}}}H\right),\quad 0<\frac{y}{h_1}<1 \quad (53)$$

$$\frac{u_2}{u_{2\max}}=f\left(\frac{y}{h_2};\frac{\rho^2\mathrm{h}_2^3 g\sin\theta}{\mu^2},\sqrt{\frac{n_3}{k_{p3}}}H,\sqrt{\frac{n_2}{k_{p2}}}h_2\right),\quad 0<\frac{y}{h_2}<1 \quad (54)$$

In which, $u_{1\max}$ and $u_{2\max}$ are the surface velocities for unplanted ground and emergent vegetation, respectively. Hereafter, we define that $\eta_1 = \dfrac{\rho^2 h_1^3 g \sin\theta}{\mu^2}$ stands for the dimensionless slope factor for unplanted ground; $\eta_2 = \sqrt{\dfrac{n_3}{k_{p3}}} H$ is the dimensionless factor of soil permeability; $\eta_3 = \dfrac{\rho^2 h_2^3 g \sin\theta}{\mu^2}$ is the dimensionless slope factor for emergent vegetation; and $\eta_4 = \sqrt{\dfrac{n_2}{k_{p2}}} h_2$ is the dimensionless factor of permeability of the vegetation layer. These four factors are discussed with respect to their effects on overland flow.

3. RESULTS AND DISCUSSIONS

From Fig. 2, flow velocity over unplanted ground increases with increasing water depth or slope for larger η_1 for a fixed η_2 without losing generality. Then, with fixed η_1, as the factor of soil permeability η_2 decreases, it means that the soil is more permeable. Thus the tangential velocity at the ground surface becomes larger as shown in Fig. 3. Furthermore, the overland flow is barely affected by larger η_2. This implies that the flow velocity will remain unchanged as the soil permeability of Region 3 becomes very small.

For vegetated ground, with fixed η_3 and η_4, the flow in Fig. 4 becomes more uniform when the grass is emergent. Also, from Fig. 4, it is very interesting to note that when the velocity distribution for $y/h_2 > 0.3$, the factor of soil permeability η_2 has almost no influence on the flow. With fixed η_2 and η_4, the trend of the effect of η_3 on the flow is similar to that of η_1 except that the flow becomes more uniform for emergent vegetation as shown in Fig. 5.

Fig. 6 shows the effect of vegetation density on the surface flow. With fixed η_2 and η_3, if the parameter η_4 increases, the grasses are planted in a denser pattern and, consequently, the flow goes more slowly due to the retarding effect of vegetation.

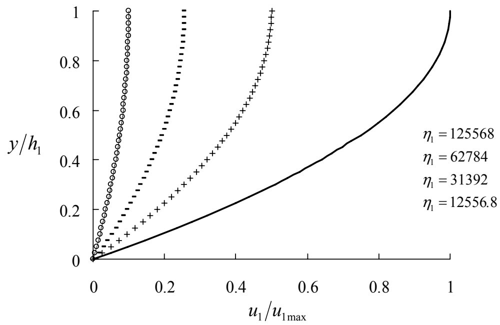


Fig.2 Effect of η_1 on vertical velocity profiles for unplanted ground

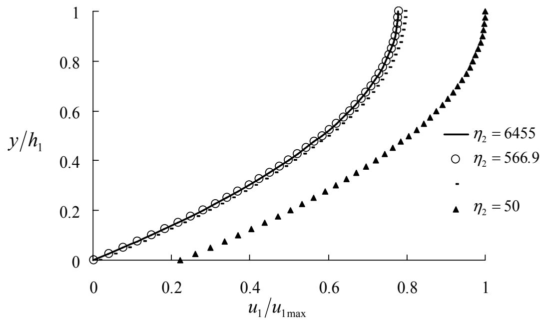


Fig.3 Effect of η_2 on vertical velocity profiles for unplanted ground

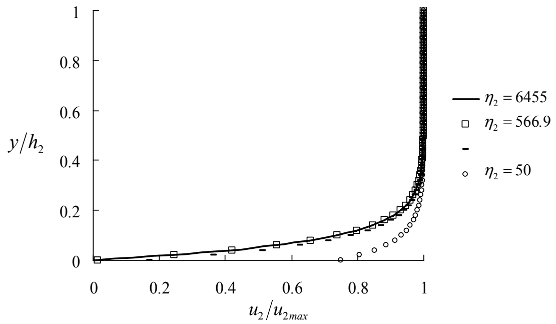


Fig.4 Effect of η_2 on vertical velocity profiles with emergent grasses

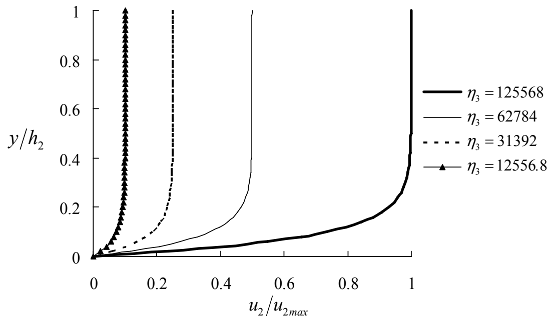


Fig.5 Effect of η_3 on vertical velocity profiles with emergent grasses

1. Darcy-Weisbach friction formula

Referring to the work of Kadlec (1990), the Darcy-Weisbach (D-W) formula can be written as

$$S = f\frac{\overline{V}^2}{8gh} \tag{55}$$

Where S = slope, $\overline{V}$ = mean velocity, h = water depth, and f = friction factor. Chen (1976) took $f = C/R_e$, C is a dimensionless parameter related to surface roughness, $R_e = \rho\overline{V}h/\mu$ is Reynolds number.

Applying the study of Gilley et al. (1985), C =200 for shallow water flowing over an unplanted ground, the mean velocity by D-W formula is

$$\overline{V_1} = \frac{\rho g \sin\theta}{25\mu}\cdot h^2 \tag{56}$$

In the case of grassed ground, C =800 while applying Helmers and Eisenhauer's work (2006), we have

$$\overline{V_2} = \frac{\rho g \sin\theta}{100\mu}\cdot h^2 \tag{57}$$

Although the D-W formula can estimate the mean velocity for unplanted ground (C =200, Eq. (56)) and grassed ground (C =800, Eq. (57)), it cannot describe the effect of different vegetation conditions.

2. Kinematic wave equation

Referring to Woolhiser and Ligget (1967) and Eagleson (1970), the overland flow by kinematic wave equation (K-W eq.) for laminar flow becomes

$$\bar{v} = (\frac{2g\sin\theta}{C_r})^{1/2} h^{1/2} \tag{58}$$

In which, $\bar{v}$ is mean velocity, and $C_r = 4/R_e$ is a function of Reynolds number. The mean velocity can be written as

$$\bar{v} = \frac{\rho g \sin\theta}{2\mu}\cdot h^2 \tag{59}$$

Eq. (59) indicates that the mean velocity from the K-W equation is related to fluid density, fluid viscosity, gravitational acceleration, slope and water depth. However, the K-W equation cannot show the influence of different surface roughness or ground covers.

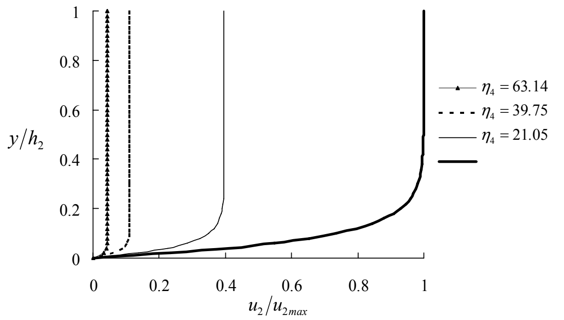


Fig.6 Effect of η_4 on vertical velocity profiles with emergent grasses

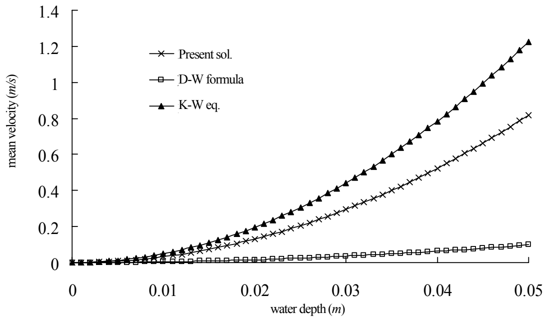


Fig.7 Mean velocity versus water depth for unplanted ground

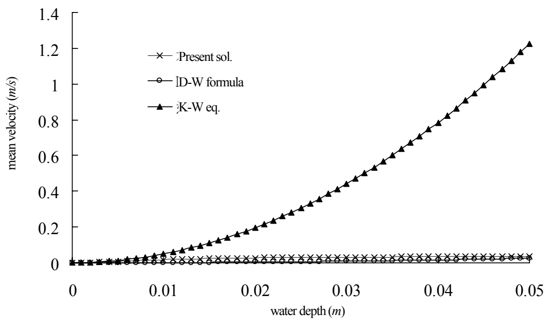


Fig. 8 Mean velocity versus water depth with emergent grasses (n_2=0.95)

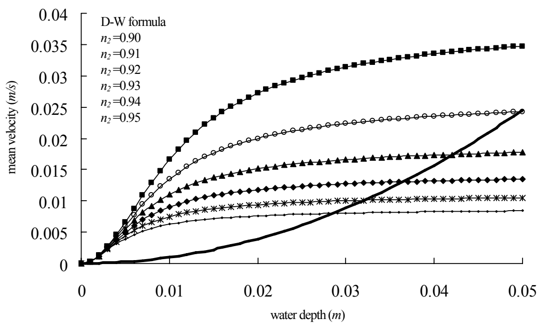


Fig.9 Mean velocity versus water depth for different vegetated density in comparison to the W-S formula

3. Comparison

Using $\rho = 1000\,kg/m^3$, $g = 9.81\,m/s^2$, $\sin\theta = 0.0001$, $\mu = 0.001\,N \cdot s/m^2$, $n_3 = 0.5$, $kp_3 = 3\times10^{-11}\,m^2$, $H = 0.05\,m$, $d_c = 0.003\,m$ and vegetation permeability k_{p2} calculated from Eq. (52), the mean velocities for unplanted ground and grassed ground are compared in Fig. 7 and Fig. 8, respectively. From Fig. 7, mean velocities for different water depths obtained by the present solution are between those by the W-S formula and the K-W equation. Fig. 8 implies that mean velocity is reduced by grasses so that it decreases very quickly as compared with that in. 7. Furthermore, as the porosity of vegetation layer increases, say n_2 =0.95, there exists a large discrepancy of the mean velocity between the present solution and the D-W formula as shown in Fig. 8. In addition, the mean velocities using the K-W equation are over-estimated for grassed ground. The comparison between the D-W formula and the present solution is shown again at a magnified scale in Fig. 9. It indicates that when n_2 =0.95 and $h_2 \le 5$ cm, the mean velocities by the present approach are larger than those given by the D-W formula.

Furthermore, employing $h_1 = h_2 = 0.04\,m$, n_2 =0.9, and $kp_2 = 8.2\times10^{-6}\,m^2$ to calculate u_1 and u_2 , the overall vertical velocity profiles for both cases are plotted in Fig. 10 and Fig. 11. From the magnified graphs, the velocities at the ground surface are not equal to zero, but they approach zero very quickly for deeper locations.

4. Shear stress distribution

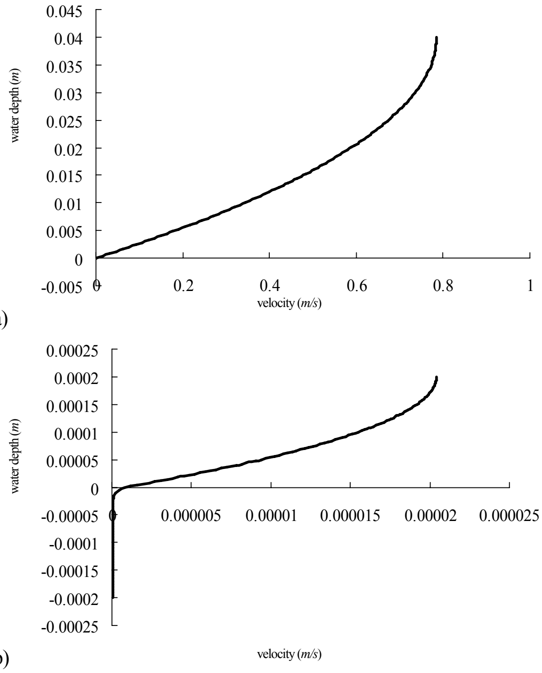
The fluid shear stress is discussed for both unplanted and grassed ground conditions. According to Newton's law of viscosity, the shear stress for Newtonian fluids is

$$\tau = \mu \frac{du_i}{dy} \tag{60}$$

In addition, from open channel flow we know

$$\tau_{\max} = \rho\, g\, h_i \sin\theta \tag{61}$$

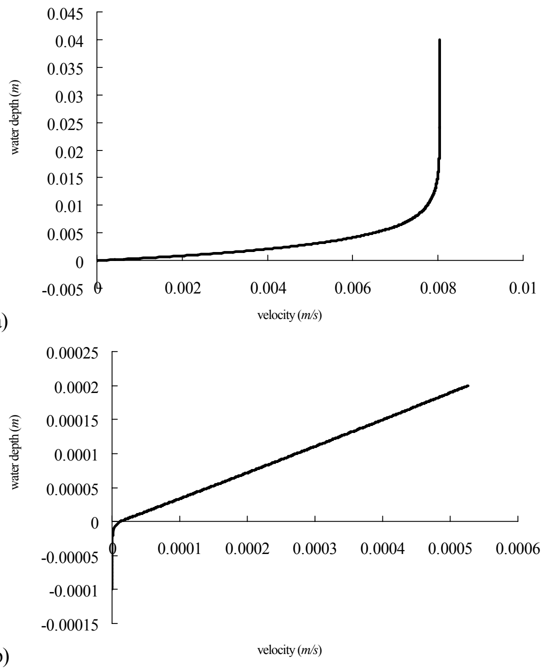
In Eqs. (60) and (61), $i = 1$ for unplanted ground and $i = 2$ for grassed ground. Fig. 12 shows that the shear stress distributions are linear and coincident with a slope of -1 in the homogenous water layer for unplanted ground, but the distributions in the soil layer are clearly affected by the factor η_2 . The phenomenon doesn't appear in Fig. 13 and both lay ers are affected by the factor η_2 for emergent vegetation. In contrast to Fig. 14, the fluid shear stress distributions in the vegetation layer are affected by the factor η_4 while they are not affected in the soil layer.



(a)

(b)

Fig.10 The overall velocity profile for unplanted ground



(a)

(b)

Fig.11 The overall velocity profile with emergent grasses

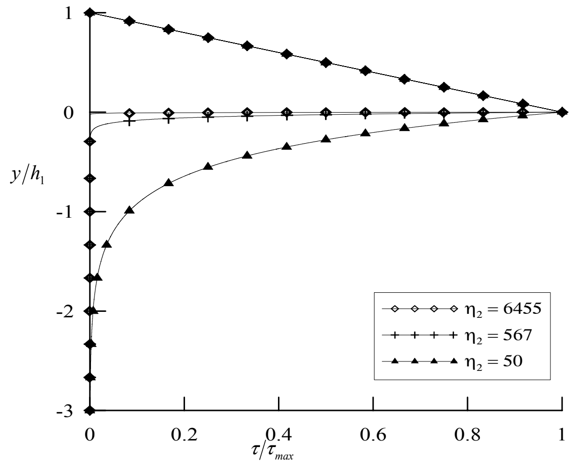


Fig.12 Effect of η_2 on shear stress distributions for unplanted ground

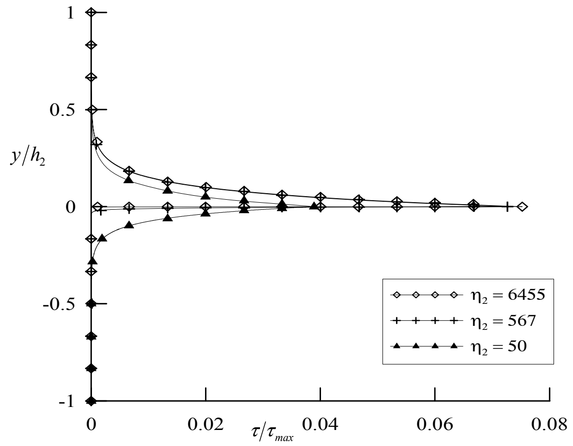


Fig.13 Effect of η_2 on shear stress distributions for grassed ground (η_4=13.25)

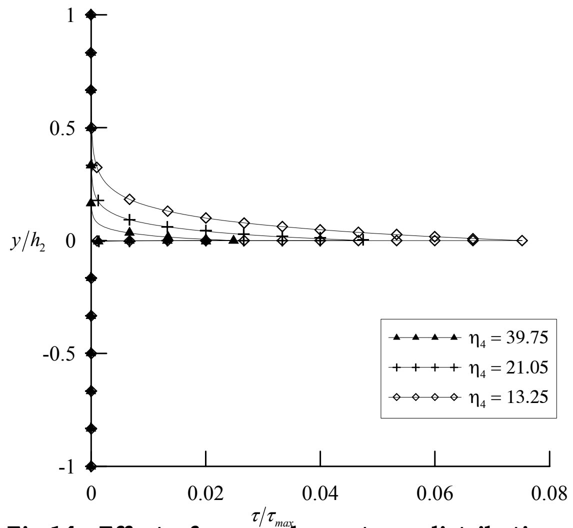


Fig.14 Effect of η_4 on shear stress distributions for grassed ground (η_2=6455)

4. Conclusions

This study proposed a new direct approach to analyze the vertical velocity distribution by applying the theory of hydraulics and poroelasticity. The relation between flow and vegetation is dominated by porous medium flow instead of the conventional view of frictional drag. As a result, a closed form solution of vertical velocity distribution and the fluid shear stress distribution were obtained. For unplanted ground, the longitudinal velocity increases with water depth and slope as expected. Although the soil permeability explains the non-zero velocity at the ground surface, its influence on the vertical velocity profile of surface flow is insignificant. For ground covered by emergent grasses, the vertical velocity profile is very close to that of uniform flow. This is owing to the fact that the surface flow is obviously retarded by the grasses, especially when the grasses are planted in a much denser pattern. The mean flow velocity is compared with that estimated by the D-W formula and the K-W approach. The results indicate that the velocity will be overestimated by the K-W approach whether the ground is unplanted or grassed. Furthermore, fluid shear stress distributions were also investigated for both unplanted and grassed ground. The results show that the shear stress distribution for the homogeneous water layer is linear with a slope of -1, and the distribution for the soil layer and the vegetation layer depends on their permeability. In conclusion, we demonstrates that flow velocities and shear stresses can be estimated more feasibly by a more systematic and theoretical approach.

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